Status Report from Tau subgroup of the HFAG

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The aim of Tau subgroup of the HFAG is to provide average values of the mass and branching fractions of the tau lepton. Using the branching fractions, we present tests of charged current lepton universality and obtain estimates for $|V_{us}|$. We also summarize the status of searches for lepton flavor violating decays of the tau lepton.

The latest averages from the HFAG have been published in Ref. [1]. Online updates of the Tau Section are available at http://www.slac.stanford.edu/xorg/hfag/tau/.

1. Mass of the τ lepton

The mass of the τ lepton has recently been measured by the BaBar [2] and Belle [3] experiments using the end-point technique from the pseudo-mass distribution in $\tau^- \to \pi^- \pi^- \pi^+ \nu_\tau$ decays, as well as by the KEDR experiment [4] from a study of the $\tau^+ \tau^-$ cross-section around the production threshold. In Figure 1, we present the measurements and average values of m_τ.

2. τ Branching Fractions:

Average values of the τ branching fractions [1] are obtained from a global unitarity constrained fit. We account for correlations due to common dependence of normalization on the τ−pair cross-section [5] and assumed branching fractions for the background modes. The detector-specific systematics uncertainties are considered to be fully correlated between measurements of the same ex-
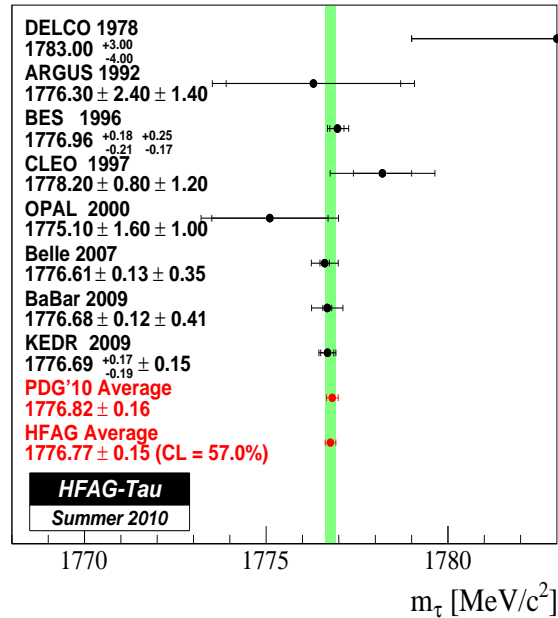

Figure 1. Measurements and average value of m_τ.

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[1]charge conjugate τ decays are implied throughout

periment.

We use 131 measurements from non-B-factory experiments, which includes the set used in the global fit performed by the PDG [6]. The measurements from non-B-factories include 37 measurements from ALEPH, 2 measurements from ARGUS, 1 measurement from CELLO, 36 measurements from CLEO, 6 measurements from CLEO3, 14 measurements from DELPHI, 2 measurements from HRS, 11 measurements from L3, 19 measurements from OPAL, and 3 measurements from TPC. Finally, we include the following measurements from the B-factories (BaBar and Belle):

- 12 measurements from BaBar:

$$\mathcal{B}(\tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau)/\mathcal{B}(\tau^- \to e^- \bar{\nu}_e \nu_\tau) \text{ [7]},$$
$$\mathcal{B}(\tau^- \to \pi^- \nu_\tau)/\mathcal{B}(\tau^- \to e^- \bar{\nu}_e \nu_\tau) \text{ [7]},$$
$$\mathcal{B}(\tau^- \to K^- \nu_\tau)/\mathcal{B}(\tau^- \to e^- \bar{\nu}_e \nu_\tau) \text{ [7]},$$
$$\mathcal{B}(\tau^- \to K^- \pi^0 \nu_\tau) \text{ [8]}$$
$$\mathcal{B}(\tau^- \to \bar{K}^0 \pi^- \nu_\tau) \text{ [9]}$$
$$\mathcal{B}(\tau^- \to \bar{K}^0 \pi^- \pi^0 \nu_\tau) \text{ [10]}$$
$$\mathcal{B}(\tau^- \to \pi^- \pi^- \pi^+ \nu_\tau \text{ (ex. } K^0)) \text{ [11]}$$
$$\mathcal{B}(\tau^- \to K^- \pi^- \pi^+ \nu_\tau \text{ (ex. } K^0)) \text{ [11]}$$
$$\mathcal{B}(\tau^- \to K^- \pi^- K^+ \nu_\tau) \text{ [11]}$$
$$\mathcal{B}(\tau^- \to K^- K^- K^+ \nu_\tau) \text{ [11]}$$
$$\mathcal{B}(\tau^- \to 3h^- 2h^+ \nu_\tau \text{ (ex. } K^0)) \text{ [12]}$$
$$\mathcal{B}(\tau^- \to 2\pi^- \pi^+ \eta \nu_\tau \text{ (ex. } K^0)) \text{ [13]}$$

- and 10 measurements from Belle:

$$\mathcal{B}(\tau^- \to h^- \pi^0 \nu_\tau) \text{ [14]}$$
$$\mathcal{B}(\tau^- \to \bar{K}^0 \pi^- \nu_\tau) \text{ [15]}$$
$$\mathcal{B}(\tau^- \to \pi^- \pi^- \pi^+ \nu_\tau \text{ (ex. } K^0)) \text{ [16]}$$
$$\mathcal{B}(\tau^- \to K^- \pi^- \pi^+ \nu_\tau \text{ (ex. } K^0)) \text{ [16]}$$
$$\mathcal{B}(\tau^- \to K^- \pi^- K^+ \nu_\tau) \text{ [16]}$$
$$\mathcal{B}(\tau^- \to K^- K^- K^+ \nu_\tau) \text{ [16]}$$
$$\mathcal{B}(\tau^- \to \pi^- \pi^0 \eta \nu_\tau) \text{ [17]}$$
$$\mathcal{B}(\tau^- \to K^- \eta \nu_\tau) \text{ [17]}$$
$$\mathcal{B}(\tau^- \to K^- \pi^0 \eta \nu_\tau) \text{ [17]}$$
$$\mathcal{B}(\tau^- \to \bar{K}^0 \pi^- \eta \nu_\tau) \text{ [17]}$$

All of these 153 measurements are expressed as a linear function of the form $\frac{(\sum_i \alpha_i P_i)}{(\sum_j \beta_j P_j)}$ of "base" branching fractions (P_i), which are chosen such that they sum up to unity. The results of the fit are shown in Table 1, which has $\chi^2/\text{ndof} = 143.4/117$ (CL = 4.9%).
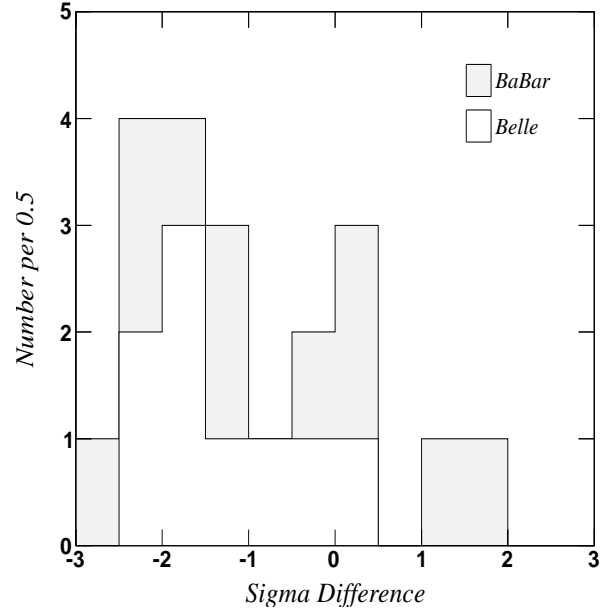

Figure 2. Normalized difference of B-factory measurements w.r.t average of non-B-factory measurements.

The differences between the global fit performed by PDG and us are:

- We expand the list of base modes from 31 to 37 to account for recent measurements by the B-factories with significant precision.

- We use the original correlation matrix published by ALEPH [18] between hadronic modes instead of between pionic modes.

- We use updated background estimates to adjust the B-factory measurements.

- We avoid applying the PDG-style scale factors to all our measurements. However, the BaBar and Belle measurements of $\mathcal{B}(\tau^- \to K^- K^+ K^- \nu_\tau)$ are 5.4 σ apart. We scale the errors on these 2 measurements only, following the PDG procedure for single-quantity averaging.

The three-kaon decays had not been observed before the B-factory era. For the remaining 20 B-factory measurements, Figure 2 shows a histogram of the normalized difference ((B-factory value minus averaged non-B-factory value)/estimated uncertainty in the difference). The average difference between the two sets of measurements is -0.98 σ (-1.26 σ for the 9 Belle measurements and -0.75 σ for the 11 BaBar measurements). Although this systematic trend is yet to be understood, the magnitude of discrepancy is less than the value of reported by the PDG [6].

3. Tests of Lepton Universality

From the unitarity constrained fit, we obtain $\mathcal{B}(\tau^- \to \mu^- \overline{\nu}_\mu \nu_\tau)/\mathcal{B}(\tau^- \to e^- \overline{\nu}_e \nu_\tau) = 0.9762 \pm 0.0028$, which includes a correlation of 18.33% between the branching fractions. This yields a value of $\left(\frac{g_\mu}{g_e}\right) = 1.0019 \pm 0.0014$, which is consistent with the Standard Model (SM) value.

Using the world averaged mass, lifetime and meson decay rates [6], we determine $\left(\frac{g_\tau}{g_\mu}\right) = 0.9966 \pm 0.0030$ (0.9860 $\pm$ 0.0073) from the pionic and kaonic branching fractions, with a correlation of 13.10%. Combining these results, we obtain $\left(\frac{g_\tau}{g_\mu}\right) = 0.9954 \pm 0.0029$, which is consistent with (1.6 σ below) the SM expectation.

We also test lepton universality between τ and μ (e), by comparing the averaged electronic (muonic) branching fractions of the τ lepton with the predicted branching fractions from measurements of the τ and μ lifetimes and their respective masses [6]. This gives $\left(\frac{g_\tau}{g_\mu}\right) = 1.0011 \pm 0.0021$ and $\left(\frac{g_\tau}{g_e}\right) = 1.0030 \pm 0.0021$. The correlation co-efficient between the determination of $\left(\frac{g_\tau}{g_\mu}\right)$ from electronic branching fraction with the ones obtained from pionic and kaonic branching frac-

Base modes (from τ^- decay)	Branching fractions (in %)
leptonic modes	
$e^- \overline{\nu}_e \nu_\tau$	17.833 ± 0.040
$\mu^- \overline{\nu}_\mu \nu_\tau$	17.408 ± 0.038
non-strange modes	
$\pi^- \nu_\tau$	10.831 ± 0.051
$\pi^- \pi^0 \nu_\tau$	25.531 ± 0.090
$\pi^- 2\pi^0 \nu_\tau$ (ex. K^0)	9.278 ± 0.097
$\pi^- 3\pi^0 \nu_\tau$ (ex. K^0)	1.046 ± 0.074
$h^- 4\pi^0 \nu_\tau$ (ex. K^0, η)	0.107 ± 0.039
$K^- K^0 \nu_\tau$	0.160 ± 0.016
$K^- \pi^0 K^0 \nu_\tau$	0.162 ± 0.019
$\pi^- K_S^0 K_S^0 \nu_\tau$	0.024 ± 0.005
$\pi^- K_S^0 K_L^0 \nu_\tau$	0.119 ± 0.024
$\pi^- \pi^- \pi^+ \nu_\tau$ (ex. K^0, ω)	8.983 ± 0.050
$\pi^- \pi^- \pi^+ \pi^0 \nu_\tau$ (ex. K^0, ω)	2.751 ± 0.069
$h^- h^- h^+ 2\pi^0 \nu_\tau$ (ex. K^0, ω, η)	0.097 ± 0.036
$h^- h^- h^+ 3\pi^0 \nu_\tau$	0.032 ± 0.003
$\pi^- K^- K^+ \nu_\tau$	0.144 ± 0.003
$\pi^- K^- K^+ \pi^0 \nu_\tau$	0.006 ± 0.002
$3h^- 2h^+ \nu_\tau$ (ex. K^0)	0.082 ± 0.003
$3h^- 2h^+ \pi^0 \nu_\tau$ (ex. K^0)	0.020 ± 0.002
$\pi^- \pi^0 \eta \nu_\tau$	0.139 ± 0.007
$\pi^- \omega \nu_\tau$	1.959 ± 0.064
$h^- \pi^0 \omega \nu_\tau$	0.409 ± 0.042
strange modes	
$K^- \nu_\tau$	0.697 ± 0.010
$K^- \pi^0 \nu_\tau$	0.431 ± 0.015
$K^- 2\pi^0 \nu_\tau$ (ex. K^0)	0.060 ± 0.022
$K^- 3\pi^0 \nu_\tau$ (ex. K^0, η)	0.039 ± 0.022
$\bar{K}^0 \pi^- \nu_\tau$	0.831 ± 0.018
$\bar{K}^0 \pi^- \pi^0 \nu_\tau$	0.350 ± 0.015
$\bar{K}^0 \pi^- 2\pi^0 \nu_\tau$	0.035 ± 0.023
$\bar{K}^0 h^- h^- h^+ \nu_\tau$	0.028 ± 0.020
$K^- \pi^- \pi^+ \nu_\tau$ (ex. K^0, ω)	0.293 ± 0.007
$K^- \pi^- \pi^+ \pi^0 \nu_\tau$ (ex. K^0, ω, η)	0.041 ± 0.014
$K^- \phi \nu_\tau (\phi \to KK)$	0.004 ± 0.001
$K^- \eta \nu_\tau$	0.016 ± 0.001
$K^- \pi^0 \eta \nu_\tau$	0.005 ± 0.001
$\bar{K}^0 \pi^- \eta \nu_\tau$	0.009 ± 0.001
$K^- \omega \nu_\tau$	0.041 ± 0.009
Sum of strange modes	2.8796 ± 0.0501
Sum of all modes	100.00

Table 1
Results of unitarity constrained fit.

tions are 48.16% and 21.82%, respectively. Averaging the three values, we obtain $\left(\frac{g_\tau}{g_\mu}\right) = 1.0001 \pm 0.0020$, which is consistent with the SM value. In Figure 3, we compare these determinations with the values obtained from pion [19], kaon [20] and W decays [21].

4. Measurement of $|V_{us}|$

Using the unitarity constraint on the first row of the CKM matrix and $|V_{ud}| = 0.97425 \pm 0.00022$ [24], one gets $|V_{us}| = 0.2255 \pm 0.0010$. Here we present 3 extractions for $|V_{us}|$ using $\mathcal{B}(\tau^- \to K^- \nu_\tau)$, $\mathcal{B}(\tau^- \to K^- \nu_\tau)/\mathcal{B}(\tau^- \to \pi^- \nu_\tau)$, and inclusive sum of τ branching fractions having net strangeness of unity in the final state.

We use the value of kaon decay constant $f_K = 157 \pm 2$MeV obtained from Lattice calculations [22], and our value of $\mathcal{B}(\tau^- \to K^- \nu_\tau) = \frac{G_F^2 f_K^2 |V_{us}|^2 m_\tau^3 \tau_\tau}{16\pi\hbar}\left(1 - \frac{m_K^2}{m_\tau^2}\right)^2 S_{EW}$, where $S_{EW} = 1.0201 \pm 0.0003$ [23], to determine $|V_{us}| = 0.2204 \pm 0.0032$, which is consistent with (1.5 σ below) the value from CKM unitarity.

We extract $|V_{us}|$ using $|V_{ud}|$, Lattice calculation of the ratio of decay constants $f_K/f_\pi = 1.189 \pm 0.007$ [22] and the long-distance correction $\delta_{LD} = (0.03 \pm 0.44)\%$, estimated using corrections to $\tau \to h\nu_\tau$ and $h \to \mu\nu_\mu$ [25], for the ratio

$$\frac{\mathcal{B}(\tau^- \to K^- \nu_\tau)}{\mathcal{B}(\tau^- \to \pi^- \nu_\tau)} = \frac{f_K^2 |V_{us}|^2}{f_\pi^2 |V_{ud}|^2}\frac{\left(1 - \frac{m_K^2}{m_\tau^2}\right)^2}{\left(1 - \frac{m_\pi^2}{m_\tau^2}\right)^2}(1+\delta_{LD}),$$

where short-distance electro-weak corrections cancel in this ratio measured to be $\mathcal{B}(\tau^- \to K^- \nu_\tau)/\mathcal{B}(\tau^- \to \pi^- \nu_\tau) = 0.0644 \pm 0.0009$. This includes a correlation of -0.49% between the branching fractions, and yields $|V_{us}| = 0.2238 \pm 0.0022$, which is also consistent with (0.7 σ below) $|V_{us}|$ from CKM unitarity.

The total hadronic width of the τ normalized to the electronic branching fraction, $R_{\mathrm{had}} = \mathcal{B}_{\mathrm{had}}/\mathcal{B}_{\mathrm{e}}$, can be written as $R_{\mathrm{had}} = R_{\mathrm{non-strange}} + R_{\mathrm{strange}}$. We can then measure

$$|V_{us}| = \sqrt{R_{\mathrm{strange}}/\left[\frac{R_{\mathrm{non-strange}}}{|V_{ud}|^2} - \delta R_{\mathrm{theory}}\right]}.$$
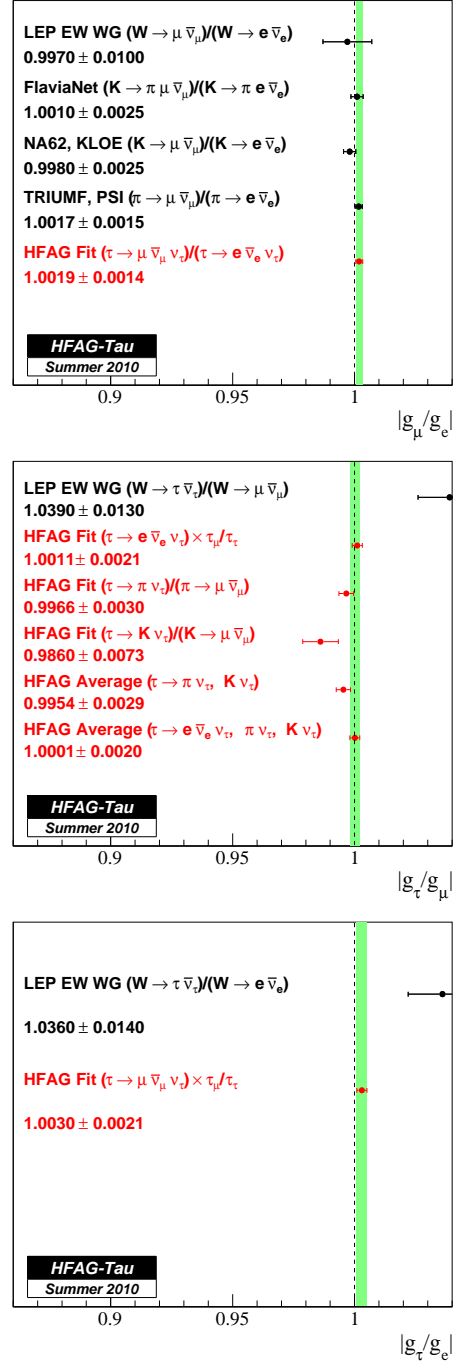

Figure 3. Measurements of lepton universality from W, kaon, pion and tau decays.

Here, we use $|V_{ud}| = 0.97425 \pm 0.00022$ [24], and $\delta R_{\text{theory}} = 0.240 \pm 0.032$ [26] which contributes to an error of 0.0010 on $|V_{us}|$. We note that this error is equivalent to half the difference between calculations of $|V_{us}|$ obtained using fixed order perturbation theory (FOPT) and contour improved perturbation theory (CIPT) calculations of δR_{theory} [27], and twice as large as the theoretical error proposed in Ref. [28].

As in Ref. [29], we improve upon the estimate of electronic branching fraction by averaging its direct measurement with its estimates of $(17.899 \pm 0.040)\%$ and $(17.794 \pm 0.062)\%$ obtained from the averaged values of muonic branching fractions and the averaged value of the lifetime of the τ lepton $= (290.6 \pm 1.0) \times 10^{-15}$ s [6], assuming lepton universality and taking into account the correlation between the leptonic branching fractions. This gives a more precise estimate for the electronic branching fraction: $\mathcal{B}_{\text{e}}^{\text{uni}} = (17.852 \pm 0.027)\%$.

Assuming lepton universality, the total hadronic branching fraction can be written as: $\mathcal{B}_{\text{had}} = 1 - 1.972558\,\mathcal{B}_{\text{e}}^{\text{uni}}$, which gives a value for the total τ hadronic width normalized to the electronic branching fraction as $R_{\text{had}} = 3.6291 \pm 0.0086$.

Non-strange width is $R_{\text{non-strange}} = R_{\text{had}} - R_{\text{strange}}$, where the value for the strange width $R_{\text{strange}} = 0.1613 \pm 0.0028$ is obtained from the sum of the strange branching fractions with the unitarity constrained fit as listed in Table 1. This gives a value of $|V_{us}| = 0.2174 \pm 0.0022$, which is $3.3\,\sigma$ lower than the CKM unitarity prediction.

Summary of these $|V_{us}|$ values are plotted in Figure 4, where we also include values from hyperon and kaon decays [20].

5. Search for lepton flavor violation in τ decays

The status of searches for lepton flavor violation in τ decays is summarized in Figure 5. A table of these results and the corresponding references are provided on the HFAG-Tau web site.
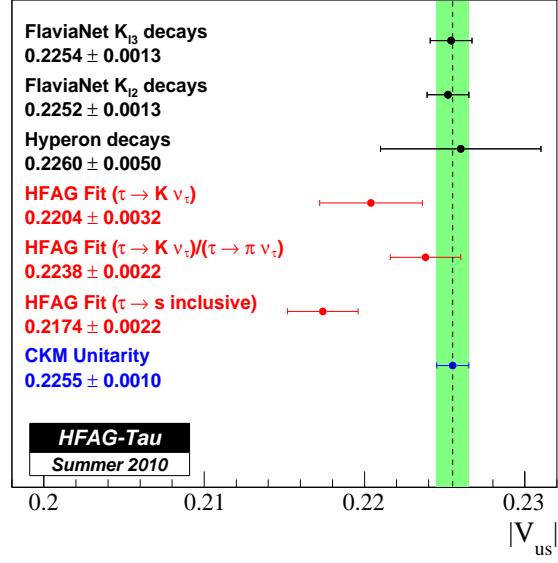

Figure 4. Measurements of $|V_{us}|$ from kaon, hyperon and tau decays.

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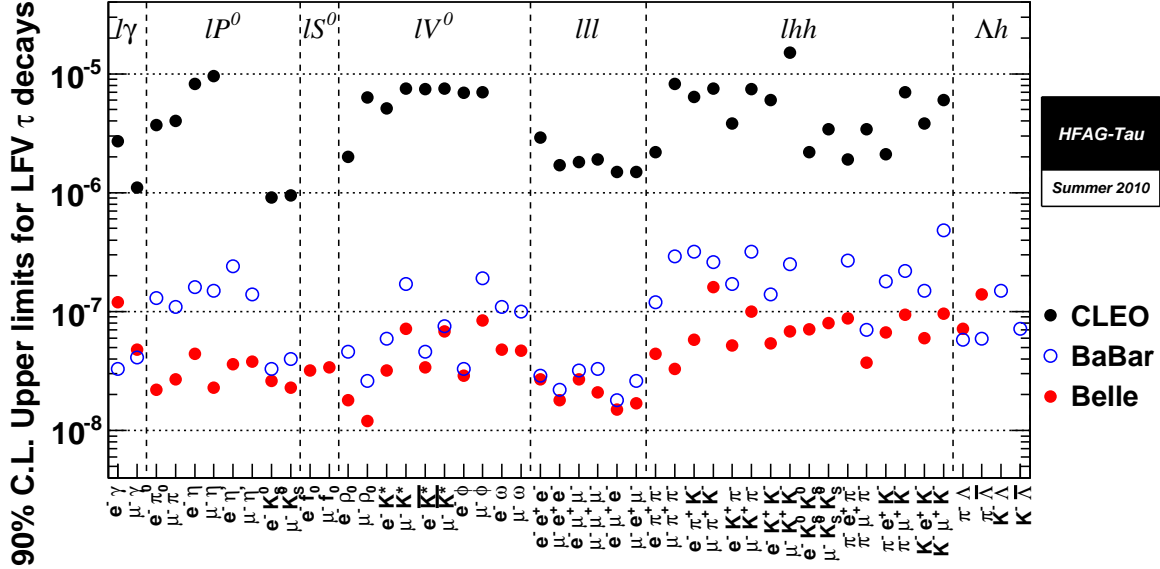
Figure 5. Status of searches for lepton flavor violation in τ decays.

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